

Now, putting all these values in the given equation (1), we get

$$u(x) + x \left[\int_0^x u(t) dt \right] + (x^2 - x) \left[\frac{1}{2} \int_0^x (x-t)^2 u(t) dt + x + 1 \right] = x e^x + 1$$

$$\Rightarrow u(x) = x e^x + 1 - x(x^2 - 1) - \int_0^x \left[x + \frac{1}{2} (x^2 - x)(x-t)^2 \right] u(t) dt$$

Which is the required integral equation

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